Guía de actividades

ECUACIONES FRACCIONARIAS DE PRIMER GRADO

Profesor Fernando Viso

GUIA DE TRABAJO

Materia: Matemáticas Guía # 55.

Tema: Ecuaciones fraccionarias de primer grado. (Baldor).

Fecha: _____

Profesor: Fernando Viso

Nombre del alumno:_____

Sección del alumno:_____

CONDICIONES:

- **Trabajo individual.**
- **Sin libros, ni cuadernos, ni notas.**
- **Sin celulares.**
- **Es obligatorio mostrar explícitamente, el procedimiento empleado para resolver cada problema.**
- **No se contestarán preguntas ni consultas de ningún tipo.**
- **No pueden moverse de su asiento. ni pedir borras, ni lápices, ni calculadoras prestadas.**

Marco Teórico:

PREGUNTAS:

Ejercicio 141. Resolver las siguientes ecuaciones:

1.- $\frac{x}{6}+5=\frac{1}{3}-x$

Solución:

m.c.m. = 6.

Se multioplican ambos lados de la igualdad por 6:

$$6\times\left(\frac{x}{6}+5\right)=6\times\left(\frac{1}{3}-x\right)\Rightarrow x+30=2-6x\Rightarrow 7x=2-30=-28\Rightarrow x=-4$$

2.- $\frac{3x}{5}-\frac{2x}{3}+\frac{1}{5}=0$

Solución:

$$15\left(\frac{3x}{5}-\frac{2x}{3}+\frac{1}{5}\right)=0\Rightarrow 9x-10x+3=0\Rightarrow x=3$$

3.- $\frac{1}{2x}+\frac{1}{4}-\frac{1}{10x}=\frac{1}{5}$

Solución:

$$20x\left(\frac{1}{2x}+\frac{1}{4}-\frac{1}{10x}\right)=20x\left(\frac{1}{5}\right)\Rightarrow 10+5x-2=4x\Rightarrow x=-8$$

4.- $\frac{x}{2}+2-\frac{x}{12}=\frac{x}{6}-\frac{5}{4}$

Solución:

$$12\left(\frac{x}{2}+2-\frac{x}{12}\right)=12\left(\frac{x}{6}-\frac{5}{4}\right)\Rightarrow 6x+24-x=2x-15\Rightarrow$$

$$\Rightarrow 3x=-39\Rightarrow x=-13$$

5.-

$$\frac{3x}{4}-\frac{1}{5}+2x=\frac{5}{4}-\frac{3x}{20}$$

Solución:

$$20\left(\frac{3x}{4}-\frac{1}{5}+2x\right)=20\left(\frac{5}{4}-\frac{3x}{20}\right)\Rightarrow 15x-4+40x=25-3x\Rightarrow$$

$$\Rightarrow 58x=29\Rightarrow x=\frac{29}{58}=\frac{1}{2}$$

6.-

$$\frac{2}{3x}-\frac{5}{x}=\frac{7}{10}-\frac{3}{2x}+1$$

Solución:

$$30x\left(\frac{2}{3x}-\frac{5}{x}\right)=30x\left(\frac{7}{10}-\frac{3}{2x}+1\right)=20-150=21x-45+30x=$$

$$=-85=51x\Rightarrow x=-\frac{85}{51}=-\frac{5}{3}$$

7.- $\frac{x-4}{3}-5=0$

Solución:

$$x-4-15=0\Rightarrow x=19$$

8.- $x-\frac{x+2}{12}=\frac{5x}{2}$

Solución:

$$12\left(x-\frac{x+2}{12}\right)=12\times\frac{5x}{2}\Rightarrow 12x-x-2=30x\Rightarrow 19x=-2\Rightarrow x=-\frac{2}{19}$$

9.- $x-\frac{5x-1}{3}=4x-\frac{3}{5}$

Solución:

$15\left(x-\frac{5x-1}{3}\right)=15\left(4x-\frac{3}{5}\right)\Rightarrow 15x-25x+5=60x-9\Rightarrow$

$\Rightarrow 70x=14\Rightarrow x=\frac{1}{5}$

10.- $10x-\frac{8x-3}{4}=2\left(x-3\right)$

Soluciuón:

$40x-8x+3=8x-24\Rightarrow 24x=-27\Rightarrow x=-\frac{27}{24}=-\frac{9}{8}$

11.- $\frac{x-2}{3}-\frac{x-8}{4}=\frac{x-4}{5}$

Solución:

$60\left(\frac{x-2}{3}-\frac{x-8}{4}\right)=60\left(\frac{x-4}{5}\right)\Rightarrow 20x-40-15x+120=12x-48\Rightarrow$

$\Rightarrow 128=7x\Rightarrow x=\frac{128}{7}$

12.- $\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}=-\frac{x-5}{5}$

Solución:

$60\left(\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}\right)=60\left(-\frac{x-5}{5}\right)\Rightarrow 30x-30-20x+40-15x+45=-12x+60\Rightarrow$

$\Rightarrow 7x=5\Rightarrow x=\frac{5}{7}$

13.- $x-\left(5x-1\right)-\frac{7-5x}{10}=1$

Solución:

$10x-10\left(5x-1\right)-7+5x=10\Rightarrow 10x-50x+10-7+5x=10\Rightarrow$

$\Rightarrow -35x=7\Rightarrow x=-\frac{1}{5}$

14.- $2x-\frac{5x-6}{4}+\frac{1}{3}\left(x-5\right)=-5x$

Solución:

$$12\left[2x-\frac{5x-6}{4}+\frac{1}{3}(x-5)\right]=12(-5x)\Rightarrow 24x-15x+18+4x-20=-60x\Rightarrow$$

$$\Rightarrow 73x=2\Rightarrow x=\frac{2}{73}$$

15.- $4-\frac{10x+1}{6}=4x-\frac{16x+3}{4}$

Solución:

$$24\left(4-\frac{10x+1}{6}\right)=24\left(4x-\frac{16x+3}{4}\right)\Rightarrow 96-40x-4=96x-96x-18\Rightarrow$$

$$\Rightarrow 110=40x\Rightarrow x=\frac{110}{40}=\frac{11}{4}$$

16.- $\frac{1}{2}(x-1)-(x-3)=\frac{1}{3}(x+3)+\frac{1}{6}$

Solución:

$$6\left[\frac{1}{2}(x-1)-(x-3)\right]=6\left[\frac{1}{3}(x+3)+\frac{1}{6}\right]\Rightarrow 3x-3-6x+18=2x+6+1\Rightarrow$$

$$\Rightarrow 8=5x\Rightarrow x=\frac{8}{5}=1\frac{3}{5}$$

17.- $\frac{6x+1}{3}-\frac{11x-2}{9}-\frac{1}{4}(5x-2)=\frac{5}{6}(6x=1)$

Solución:

$$36\left[\frac{6x+1}{3}-\frac{11x-2}{9}-\frac{1}{4}(5x-2)\right]=36\times\frac{5}{6}(6x+1)\Rightarrow 72x+12-44x+8-45x+18=180x+30\Rightarrow$$

$$\Rightarrow 8=197x\Rightarrow x=\frac{8}{197}$$

18.- $\frac{4x+1}{3}=\frac{1}{3}(4x-1)-\frac{13+2x}{6}-\frac{1}{2}(x-3)$

Solución:

$$6\left[\frac{4x+1}{3}\right]=6\left[\frac{1}{3}(4x-1)-\frac{13+2x}{6}-\frac{1}{2}(x-3)\right]\Rightarrow 8x+2=8x-2-13-2x-3x+9\Rightarrow$$

$$\Rightarrow 5x=-8\Rightarrow x=-\frac{8}{5}$$

19.- $\frac{2}{5}(5x-1)+\frac{3}{10}(10x-3)=-\frac{1}{2}(x-2)-\frac{6}{5}$

Solución:

$$10\left[\frac{2}{5}(5x-1)+\frac{3}{10}(10x-3)\right]=10\left[-\frac{1}{2}(x-2)-\frac{6}{5}\right]\Rightarrow 20x-4+30x-9=-5x+10-12\Rightarrow$$

$$\Rightarrow 55x=11\Rightarrow x=\frac{1}{5}$$

20.- $\frac{3x-1}{2}-\frac{5x+4}{3}-\frac{x+2}{8}=\frac{2x-3}{5}-\frac{1}{10}$

Solución:

$$120\left[\frac{3x-1}{2}-\frac{5x+4}{3}-\frac{x+2}{8}\right]=120\left[\frac{2x-3}{5}-\frac{1}{10}\right]\Rightarrow$$

$$\Rightarrow 180x-60-200x-160-15x-30=48x-72-12\Rightarrow$$

$$\Rightarrow -166=83x\Rightarrow x=-2$$

21.- $\frac{7x-1}{3}-\frac{5-2x}{2x}=\frac{4x-3}{4}+\frac{1+4x^2}{3x}$

Solución:

$$12x\left[\frac{7x-1}{3}-\frac{5-2x}{2x}\right]=12x\left[\frac{4x-3}{4}+\frac{1+4x^2}{3x}\right]\Rightarrow 28x^2-4x-30+12x=12x^2-9x+4+16x^2\Rightarrow$$

$$\Rightarrow 17x=34\Rightarrow x=2$$

22.- $\frac{2x+7}{3}-\frac{2\left(x^2-4\right)}{5x}-\frac{4x^2-6}{15x}=\frac{7x^2+6}{3x^2}$

Solución:

$$15x^2\left[\frac{2x+7}{3}-\frac{2\left(x^2-4\right)}{5x}-\frac{4x^2-6}{15x}\right]=15x^2\left(\frac{7x^2+6}{3x^2}\right)\Rightarrow$$

$$\Rightarrow 10x^3+35x^2-6x^3+24x-4x^3+6x=35x^2+30\Rightarrow$$

$$\Rightarrow 30x=30\Rightarrow x=1$$

23.- $\frac{2}{3}\left(\frac{x+1}{5}\right)=\frac{3}{4}\left(\frac{x-6}{3}\right)$

Solución:

$$\frac{2}{15}(x+1)=\frac{1}{4}(x-6)\Rightarrow 60\left[\frac{2}{15}(x+1)\right]=60\left[\frac{1}{4}(x-6)\right]\Rightarrow$$

$$\Rightarrow 8(x+1)=15(x-6)\Rightarrow 8x+8=15x-90\Rightarrow 98=7x\Rightarrow x=\frac{98}{7}=14$$

24.- $\frac{3}{5}\left(\frac{2x-1}{6}\right)-\frac{4}{3}\left(\frac{3x+2}{4}\right)-\frac{1}{5}\left(\frac{x-2}{3}\right)+\frac{1}{5}=0$

Solución:

$$\frac{2x-1}{10}-\frac{3x+2}{3}-\frac{(x-2)}{15}+\frac{1}{5}=0\Rightarrow 30\left(\frac{2x-1}{10}-\frac{3x-2}{3}-\frac{(x-2)}{15}+\frac{1}{5}\right)\Rightarrow$$

$$\Rightarrow 6x-3-30x-20-2x+4+6=0\Rightarrow -26x=13\Rightarrow x=-\frac{1}{2}$$

25.- $10-\frac{3x+5}{6}=3\frac{11}{16}-\frac{\frac{x}{2}}{4}$

Solución:

$$10-\frac{3x+5}{6}=\frac{47}{12}-\frac{x}{8}\Rightarrow 72\left(10-\frac{(3x+5)}{6}\right)=72\left(\frac{47}{12}-\frac{x}{8}\right)\Rightarrow$$

$$\Rightarrow 720-36x-60=282-9x\Rightarrow 660-282=36x-9x\Rightarrow$$

$$\Rightarrow 378=27x\Rightarrow x=\frac{378}{27}=14$$

26.- $9x-2-7x\left(\frac{1}{x}-\frac{1}{2}\right)=\frac{1+\frac{x}{2}}{2}+2\frac{3}{4}$

Solución:

$$9x-2-7x\left(\frac{2-x}{2x}\right)=\frac{\frac{2+x}{2}}{2}+\frac{11}{4}\Rightarrow 9x-2-7\left(\frac{2-x}{2}\right)=\frac{2+x}{4}+\frac{11}{4}\Rightarrow$$

$$\Rightarrow 4\left[9x-2-7\left(\frac{2-x}{2}\right)\right]=4\left[\left(\frac{2+x}{4}\right)+\frac{11}{4}\right]\Rightarrow 36x-8-28+14x=2+x+11\Rightarrow$$

$$\Rightarrow 49x=49\Rightarrow x=\frac{49}{49}=1$$

27.- $\frac{3x}{8}-\frac{7}{10}-\frac{12x-5}{16}-\frac{2x-3}{20}+\frac{4x+9}{4}+\frac{7}{80}=$

Solución:

m.c.m. = 80.

$$80\left[\frac{3x}{8}-\frac{7}{10}-\frac{12x-5}{16}-\frac{2x-3}{20}+\frac{4x+9}{4}+\frac{7}{80}\right]=0\Rightarrow$$
$$\Rightarrow 30x-56-60x+25-8x+12+80x+180+7=0\Rightarrow$$
$$\Rightarrow 42x+168=0\Rightarrow x=-\frac{168}{42}=-4$$

28.- $\frac{5x}{4}-\frac{3}{17}(x-20)-(2x-1)=\frac{x+24}{34}$

m.c.m. de los denominadores = 68.

$$68\left[\frac{5x}{4}-\frac{3}{17}(x-20)-(2x-1)\right]=68\left[\frac{x+24}{34}\right]\Rightarrow$$
$$\Rightarrow 85x-12x+240-136x+68=2x+48\Rightarrow 240+68-48=2x+12x+136x-85x\Rightarrow$$
$$\Rightarrow 260=65x\Rightarrow x=\frac{260}{65}=\frac{52}{13}=4$$

29.- $5+\frac{x}{4}=\frac{1}{3}\left(2-\frac{x}{2}\right)-\frac{2}{3}+\frac{1}{4}\left(10-\frac{5x}{3}\right)$

Solución:

$$\frac{20+x}{4}=\frac{4-x}{6}-\frac{2}{3}+\frac{30-5x}{12}\Rightarrow 12\left(\frac{20+x}{4}\right)=12\left[\left(\frac{4-x}{6}\right)-\frac{2}{3}+\frac{30-5x}{12}\right]\Rightarrow$$
$$\Rightarrow 60+3x=8-2x-8+30-5x\Rightarrow 10x=30-60=-30\Rightarrow x=-3$$

30.- $\frac{5(x+2)}{12}+\frac{4}{9}-\frac{22-x}{36}=3x-20-\frac{8-x}{12}-\frac{20-3x}{18}$

Solución:

Continúa en la próxima página

$$36\left[\frac{5(x+2)}{12}+\frac{4}{9}-\frac{22-x}{36}\right]=36\left[3x-20-\frac{8-x}{12}-\frac{20-3x}{18}\right]\Rightarrow$$

$$\Rightarrow 15(x+2)+16-22+x=108x-720-24+3x-40+6x\Rightarrow$$

$$\Rightarrow 15x+30+16-22+x=108x-720-24+3x-40+6x\Rightarrow$$

$$\Rightarrow 30+16-22+720+24+40=108x+3x+6x-15x-x\Rightarrow$$

$$\Rightarrow 808=101x\Leftrightarrow x=\frac{808}{101}=8$$

31.- $\left(3-\frac{x}{2}\right)-\left(1-\frac{x}{3}\right)=7-\left(x-\frac{x}{2}\right)$

Solución:

$$\left(\frac{6-x}{2}\right)-\left(\frac{3-x}{3}\right)=7-\left(\frac{2x-x}{2}\right)\Rightarrow 6\left[\left(\frac{6-x}{2}\right)-\left(\frac{3-x}{3}\right)\right]=42-6\left(\frac{x}{2}\right)\Rightarrow$$

$$\Rightarrow 18-3x-6+2x=42-3x\Rightarrow 2x=42-18+6=30\Rightarrow x=\frac{30}{2}=15$$

32.- $(x+3)(x-3)-x^2-\frac{5}{4}=\left(x-\frac{x}{5}\right)-\left(3x-\frac{3}{4}\right)$

Solución:

$$x^2-9-x^2-\frac{5}{4}=\left(\frac{5x-x}{5}\right)-\left(\frac{12x-3}{4}\right)\Rightarrow -9-\frac{5}{4}=\left(\frac{4x}{5}\right)-\left(\frac{12x-3}{4}\right)\Rightarrow$$

$$\Rightarrow -\frac{41}{4}=\left(\frac{4x}{5}\right)-\left(\frac{12x-3}{4}\right)\Rightarrow -20\times\frac{41}{4}=20\left[\frac{4x}{5}-\left(\frac{12x-3}{4}\right)\right]\Rightarrow$$

$$\Rightarrow -205=16x-60x+15\Rightarrow -220=-44x\Rightarrow x=\frac{220}{44}=5$$

33.- $2x-\left(2x-\frac{3x-1}{8}\right)=\frac{2}{3}\left(\frac{x+2}{6}\right)-\frac{1}{4}$

Solución:

$$2x-\left(\frac{16x-3x+1}{8}\right)=\frac{2x+4}{18}-\frac{1}{4}\Rightarrow 144\left[2x-\frac{13x+1}{8}\right]=144\left[\frac{2x+4}{18}-\frac{1}{4}\right]\Rightarrow$$

$$\Rightarrow 288x-18(13x+1)=8(2x+4)-36\Rightarrow 288x-234x-18=16x+32-36\Rightarrow$$

$$\Rightarrow 38x=14\Rightarrow x=\frac{14}{38}=\frac{7}{19}$$

Ejercicio 142. Resolver las siguientes ecuaciones de primer grado con denominadores compuestos.

1.- $\frac{3}{5}+\frac{3}{2x-1}=0$

Solución:

$$\frac{3}{5}=-\frac{3}{2x-1}=\frac{3}{1-2x}\Rightarrow 3(1-2x)=15\Rightarrow 3-6x=15\Rightarrow$$
$$\Rightarrow -6x=12\Rightarrow x=-2$$

2.- $\frac{2}{4x-1}=\frac{3}{4x+1}$

Solución:

$$2(4x+1)=3(4x-1)\Rightarrow 8x+2=12x-3\Rightarrow 5=4x\Rightarrow x=\frac{5}{4}$$

3.- $\frac{5}{x^2-1}=\frac{1}{x-1}$

Solución:

$$5(x-1)=(x+1)(x-1)\Rightarrow 5=x+1\Rightarrow x=5-1\Rightarrow x=4$$

4.- $\frac{3}{x+1}-\frac{1}{x^2-1}=0$

Solución:

$$\frac{3}{x+1}=\frac{1}{(x+1)(x-1)}\Rightarrow 3(x-1)(x+1)=(x+1)\Rightarrow 3(x-1)=1\Rightarrow$$
$$\Rightarrow 3x-3=1\Rightarrow 3x=4\Rightarrow x=\frac{4}{3}$$

5.- $\frac{5x+8}{3x+4}=\frac{5x+2}{3x-4}$

Solución:

$$(5x+8)(3x-4)=(3x+4)(5x+2)\Rightarrow 15x^2-20x+24x-32=15x^2+6x+20x+8\Rightarrow$$
$$\Rightarrow 15x^2+4x-32=15x^2+26x+8\Rightarrow -32-8=26x-4x\Rightarrow$$
$$\Rightarrow -40=22x\Rightarrow x=-\frac{40}{22}=-\frac{20}{11}$$

6.- $\dfrac{10x^2-5x+8}{5x^2+9x-19}=2$

Solución:

$10x^2-5x+8=2\left(5x^2+9x-19\right)=10x^2+18x-38\Rightarrow$

$\Rightarrow 8+38=18x+5x\Rightarrow 46=23x\Rightarrow x=\dfrac{46}{23}=2$

7.- $\dfrac{1}{3x-3}+\dfrac{1}{4x+4}=\dfrac{1}{12x-12}$

Solución:

$$\frac{1}{3(x-1)}+\frac{1}{4(x+1)}=\frac{1}{12(x-1)}$$

Solución:

$$12(x+1)(x-1)\left[\frac{1}{3(x-1)}+\frac{1}{4(x+1)}\right]=12(x+1)(x-1)\left[\frac{1}{12(x-1)}\right]\Rightarrow$$

$$\Rightarrow 4(x+1)+3(x-1)=x+1\Rightarrow 4x+4+3x-3=x+1\Rightarrow$$

$$\Rightarrow 6x=0\Rightarrow x=0$$

8.- $\dfrac{x}{4}-\dfrac{x^2-8x}{4x-5}=\dfrac{7}{4}$

Solución:

$$\frac{x(4x-5)-4\left(x^2-8x\right)}{4(4x-5)}=\frac{7}{4}\Rightarrow\frac{4x^2-5x-4x^2+32x}{4(4x-5)}=\frac{7}{4}\Rightarrow$$

$$\Rightarrow\frac{27x}{4x-5}=7\Rightarrow 27x=28x-35=28x-27x=35\Rightarrow x=35$$

9.- $\dfrac{2x-9}{10}+\dfrac{2x-3}{2x-1}=\dfrac{x}{5}$

Solución:

$$\frac{(2x-9)(2x-1)+10(2x-3)}{10(2x-1)}=\frac{x}{5}\Rightarrow\frac{4x^2-2x-18x+9+20x-30}{10(2x-1)}=\frac{x}{5}\Rightarrow$$

$$\Rightarrow 5(4x^2-21)=10x(2x-1)\Rightarrow 20x^2-105=20x^2-10x\Rightarrow$$

$$\Rightarrow 10x=105\Rightarrow x=\frac{105}{10}=\frac{21}{2}=10\frac{1}{2}$$

10.- $\dfrac{(3x-1)^2}{x-1}=\dfrac{18x-1}{2}$

Solución:

$$2(3x-1)^2=(x-1)(18x-1)\Rightarrow 2(9x^2-6x+1)=18x^2-x-18x+1\Rightarrow$$

$$\Rightarrow 18x^2-12x+2=18x^2-19x+1\Rightarrow 19x-12x=1-2\Rightarrow x=-\frac{1}{7}$$

11.- $\dfrac{2x+7}{5x+2}-\dfrac{2x-1}{5x-4}=0$

Solución:

$$\frac{2x+7}{5x+2}=\frac{2x-1}{5x-4}\Rightarrow(2x+7)(5x-4)=(5x+2)(2x-1)\Rightarrow$$

$$\Rightarrow 10x^2-8x+35x-28=10x^2+4x-5x-2\Rightarrow 28x=26\Rightarrow x=\frac{26}{28}=\frac{13}{14}$$

12.- $\dfrac{(5x-2)(7x+3)}{7x(5x-1)}-1=0$

Solución:

$$\frac{(5x-2)(7x+3)}{7x(5x-1)}=1\Rightarrow(5x-2)(7x+3)=7x(5x-1)\Rightarrow$$

$$\Rightarrow 35x^2+15x-14x-6=35x^2-7x\Rightarrow 8x=6\Rightarrow x=\frac{6}{8}=\frac{3}{4}$$

13.- $\dfrac{3}{x-4}=\dfrac{2}{x-3}+\dfrac{8}{x^2-7x+12}$

Solución:

$$\frac{3}{x-4}=\frac{2}{x-3}+\frac{8}{(x-3)(x-4)}\Rightarrow(x-3)(x-4)\left(\frac{3}{x-4}\right)=(x-3)(x-4)\left[\frac{2}{x-3}+\frac{8}{(x-3)(x-4)}\right]\Rightarrow$$
$$\Rightarrow 3(x-3)=2(x-4)+8\Rightarrow 3x-9=2x-8+8\Rightarrow 3x-2x=9\Rightarrow x=9$$

14.- $\frac{6x-1}{18}-\frac{3(x+2)}{5x-6}=\frac{1+3x}{9}$

Solución:

$$\frac{(6x-1)(5x-6)-18(3x+6)}{18(5x-6)}=\frac{1+3x}{9}\Rightarrow\frac{30x^2-36x-5x+6-54x-108}{2(5x-6)}=1+3x\Rightarrow$$
$$\Rightarrow 30x^2-95x-102=(10x-12)(3x+1)=30x^2+10x-36x-12=30x^2-26x-12\Rightarrow$$
$$\Rightarrow -69x=102-12=90\Rightarrow x=-\frac{90}{69}=-\frac{30}{23}=-1\frac{7}{23}$$

15.- $\frac{5}{1+x}-\frac{3}{1-x}-\frac{6}{1-x^2}=0$

Solución:

$$\frac{5}{1+x}-\frac{3}{1-x}=\frac{6}{(1+x)(1-x)}\Rightarrow\frac{5(1-x)-3(1+x)}{(1+x)(1-x)}=\frac{6}{(1+x)(1-x)}\Rightarrow$$
$$\Rightarrow 5-5x-3-3x=6\Rightarrow 2-8x=6\Rightarrow -8x=4\Rightarrow x=-\frac{1}{2}$$

16.- $\frac{1+2x}{1+3x}-\frac{1-2x}{1-3x}=-\frac{3x-14}{1-9x^2}$

Solución:

$$\frac{(1+2x)(1-3x)-(1+3x)(1-2x)}{1-9x^2}=-\frac{3x-14}{1-9x^2}\Rightarrow$$
$$\Rightarrow 1-3x+2x-6x^2-\left(1-2x+3x-6x^2\right)=14-3x\Rightarrow$$
$$\Rightarrow 1-x-6x^2-1-x+6x^2=14-3x\Rightarrow x=14$$

17.- $\frac{3x-1}{x^2+7x+12}=\frac{1}{2x+6}+\frac{7}{6x+24}$

Solución:

$$\frac{3x-1}{(x+3)(x+4)}=\frac{1}{2(x+3)}+\frac{7}{6(x+4)}$$

Solución:

Se debe multiplicar ambos miembros de la igualdad por $6(x+3)(x+4)$

$$6(3x-1)=3(x+4)+7(x+3)\Rightarrow 18x-6=3x+12+7x+21$$

$$\Rightarrow 8x=39\Rightarrow x=\frac{39}{8}=4\frac{7}{8}$$

18.- $\frac{1}{(x-1)^2}-\frac{3}{2x-2}=-\frac{3}{2x+2}$

Solución:

$$\frac{1}{(x-1)^2}-\frac{3}{2(x-1)}=-\frac{3}{2(x+1)}\Rightarrow \frac{2-3(x-1)}{2(x-1)^2}=-\frac{3}{2(x+1)}\Rightarrow$$

$$\Rightarrow \frac{5-3x}{2(x-1)^2}=-\frac{3}{2(x+1)}\Rightarrow (5-3x)(x+1)=-3(x-1)^2\Rightarrow$$

$$\Rightarrow 5x+5-3x^2-3x=-3x^2+6x-3\Rightarrow -4x=-8\Rightarrow x=2$$

19.- $\frac{5x+13}{15}-\frac{4x+5}{5x-15}=\frac{x}{3}$

Solución:

$$\frac{5x+13}{15}-\frac{4x+5}{5(x-3)}=\frac{x}{3}\Rightarrow \frac{(5x+13)(x-3)-3(4x+5)}{15(x-3)}=\frac{x}{3}\Rightarrow$$

$$\Rightarrow \frac{5x^2-15x+13x-39-12x-15}{15(x-3)}=\frac{x}{3}\Rightarrow \frac{5x^2-14x-54}{15(x-3)}=\frac{x}{3}\Rightarrow$$

$$\Rightarrow 15x^2-42x-162=15x^2-45x\Rightarrow 3x=162\Rightarrow x=\frac{162}{3}=54$$

20.- $\frac{2x-1}{2x+1}-\frac{x-4}{3x-2}=\frac{2}{3}$

Solución:

$$\frac{(2x-1)(3x-2)-(2x+1)(x-4)}{(2x+1)(3x-2)}=\frac{2}{3}\Rightarrow$$

$$\Rightarrow\frac{6x^2-4x-3x+2-2x^2+8x-x+4}{(2x+1)(3x-2)}=\frac{2}{3}\Rightarrow\frac{4x^2+6}{(2x+1)(3x-2)}=\frac{2}{3}\Rightarrow$$

$$12x^2+18=2\left(6x^2-4x+3x-2\right)=12x^2-2x-4\Rightarrow x=-\frac{22}{2}=-11$$

21.- $\frac{4x+3}{2x-5}-\frac{3x+8}{3x-7}=1$

Solución:

$$(4x+3)(3x-7)-(2x-5)(3x+8)=(2x-5)(3x-7)\Rightarrow$$

$$\Rightarrow 12x^2-28x+9x-21-6x^2-16x+15x+40=6x^2-14x-15x+35\Rightarrow$$

$$\Rightarrow 9x=16\Rightarrow x=\frac{16}{9}=1\frac{7}{9}$$

22.- $\frac{10x-7}{15x+3}=\frac{3x+8}{12}-\frac{5x^2-4}{20x+4}$

Solución:

$$\frac{10x-7}{3(5x+1)}+\frac{5x^2-4}{4(5x+1)}=\frac{3x+8}{12}\Rightarrow\frac{4(10x-7)+3\left(5x^2-4\right)}{12(5x+1)}=\frac{3x+8}{12}\Rightarrow$$

$$\Rightarrow\frac{40x-28+15x^2-12}{12(5x+1)}=\frac{3x+8}{12}\Rightarrow\frac{15x^2+40x-40}{5x+1}=3x+8\Rightarrow$$

$$\Rightarrow 15x^2+40x-40=15x^2+40x+3x+8\Rightarrow -48=3x\Rightarrow x=-\frac{48}{3}=-16$$

23.- $\frac{4x-1}{5}+\frac{x-2}{2x-7}=\frac{8x-3}{10}-1\frac{3}{10}$

Solución:

$$\frac{4x-1}{5}+\frac{x-2}{2x-7}=\frac{8x-3}{10}-\frac{13}{10}\Rightarrow\frac{x-2}{2x-7}=\frac{8x-3}{10}-\frac{13}{10}-\frac{2(4x-1)}{10}\Rightarrow$$

$$\Rightarrow\frac{x-2}{2x-7}=\frac{8x-3-13-8x+2}{10}=\frac{-14}{10}=-\frac{7}{5}\Rightarrow 5(x-2)=7(7-2x)\Rightarrow$$

$$\Rightarrow 5x-10=49-14x\Rightarrow 19x=59\Rightarrow x=\frac{59}{19}=3\frac{2}{19}$$

24.- $\dfrac{1}{x-1}-\dfrac{2}{x-2}=\dfrac{3}{2x-2}-\dfrac{2\frac{1}{3}}{2x-4}$

Solución:

$$\frac{1}{x-1}-\frac{2}{x-2}=\frac{3}{2(x-1)}-\frac{\frac{7}{3}}{2(x-2)}\Rightarrow\frac{1}{x-1}-\frac{2}{x-2}=\frac{3}{2(x-1)}-\frac{7}{6(x-2)}\Rightarrow$$

$$\Rightarrow 6(x-1)(x-2)\left[\frac{1}{x-1}-\frac{2}{x-2}\right]=6(x-1)(x-2)\left[\frac{3}{2(x-1)}-\frac{7}{6(x-2)}\right]=$$

$$\Rightarrow 6(x-2)-12(x-1)=9(x-2)-7(x-1)\Rightarrow 6x-12-12x+12=9x-18-7x+7\Rightarrow$$

$$\Rightarrow -8x=-11\Rightarrow x=\frac{11}{8}=1\frac{3}{8}$$

25.- $\dfrac{1}{x+3}-\dfrac{2}{5x-20}=\dfrac{1\frac{1}{2}}{3x-12}-\dfrac{2}{x+3}$

Solución:

$$\frac{1}{x+3}-\frac{2}{5(x-4)}=\frac{\frac{3}{2}}{3(x-4)}-\frac{2}{x+3}\Rightarrow\frac{1}{x+3}+\frac{2}{x+3}=\frac{3}{6(x-4)}+\frac{2}{5(x-4)}\Rightarrow$$

$$\Rightarrow\frac{3}{x+3}=\frac{15+12}{30(x-4)}=\frac{27}{30(x-4)}=\frac{9}{10(x-4)}\Rightarrow 3\times 10(x-4)=9(x+3)\Rightarrow$$

$$\Rightarrow 30x-120=9x+27\Rightarrow 21x=147\Rightarrow x=\frac{147}{21}=7$$

26.- $\dfrac{1}{6-2x}-\dfrac{4}{5-5x}=\dfrac{10}{12-4x}-\dfrac{3}{10-10x}$

Solución:

$$\frac{1}{2(3-x)}-\frac{4}{5(1-x)}=\frac{10}{4(3-x)}-\frac{3}{10(1-x)}$$

El m.c.m. de los denominadores es: $20(3-x)(1-x)$; entonces, multiplicamos ambos lados de la igualdad por el m.c.m.:

$$20(3-x)(1-x)\left[\frac{1}{2(3-x)}-\frac{4}{5(1-x)}\right]=20(3-x)(1-x)\left[\frac{10}{4(3-x)}-\frac{3}{10(1-x)}\right]\Rightarrow$$

$$\Rightarrow 10(1-x)-16(3-x)=50(1-x)-6(3-x)\Rightarrow$$

$$\Rightarrow 10-10x-48+16x=50-50x-18+6x\Rightarrow 50x=70\Rightarrow x=\frac{70}{50}=\frac{7}{5}=1\frac{2}{5}$$

27.- $\frac{2}{3}-\frac{6x^2}{9x^2-1}=\frac{2}{3x-1}$

Solución:

$$\frac{2}{3}-\frac{6x^2}{(3x+1)(3x-1)}=\frac{2}{3x-1}$$

El m.c.m. de todos los denominadores es: $3(3x+1)(3x-1)$; entonces:

$$3(3x+1)(3x-1)\left[\frac{2}{3}-\frac{6x^2}{(3x+1)(3x-1)}\right]=3(3x+1)(3x-1)\left(\frac{2}{3x-1}\right)\Rightarrow$$

$$\Rightarrow 2(3x+1)(3x-1)-18x^2=6(3x+1)\Rightarrow 18x^2-2-18x^2=18x+6\Rightarrow$$

$$\Rightarrow -2=18x+6\Rightarrow -8=18x\Rightarrow x=-\frac{4}{9}$$

28.- $\frac{5x^2-27x}{5x+3}-\frac{1}{x}=x-6$

Solución:

El m.c.m. de los denominadores es: $x(5x+3)$; entonces:

$$x(5x+3)\left[\frac{5x^2-27x}{5x+3}-\frac{1}{x}\right]=x(5x+3)(x-6)\Rightarrow$$

$$\Rightarrow 5x^3-27x^2-5x-3=5x^3-30x^2+3x^2-18x\Rightarrow$$

$$\Rightarrow 18x-5x=3\Rightarrow 13x=3\Rightarrow x=\frac{3}{13}$$

29.- $\frac{4x+1}{4x-1}-\frac{6}{16x^2-1}=\frac{4x-1}{4x+1}$

Solución:

m.c.m. de todos los denominadores: $16x^2-1=(4x+1)(4x-1)$; entonces:

$$(4x+1)(4x-1)\left[\frac{4x+1}{4x-1}-\frac{6}{(4x+1)(4x-1)}\right]=(4x+1)(4x-1)\left(\frac{4x-1}{4x+1}\right)\Rightarrow$$

$$\Rightarrow(4x+1)^2-6=(4x-1)^2\Rightarrow 16x^2+8x+1-6=16x^2-8x+1\Rightarrow$$

$$\Rightarrow 16x=6\Rightarrow x=\frac{6}{16}=\frac{3}{8}$$

30.- $3\left(\frac{x-1}{x+1}\right)+2\left(\frac{x+1}{x-4}\right)=\frac{5x(x-1)}{x^2-3x-4}$

Solución:

$$\frac{3(x-1)}{x+1}+\frac{2(x+1)}{x-4}=\frac{5x(x-1)}{(x+1)(x-4)}$$

Entonces:

$$\frac{3(x-1)(x-4)+2(x+1)^2}{(x+1)(x-4)}=\frac{5x(x-1)}{(x+1)(x-4)}$$

Si los denominadores son iguales, los numeradores tienen que ser iguales:

$$3(x^2-5x+4)+2(x^2+2x+1)=5x^2-5x\Rightarrow$$

$$\Rightarrow 3x^2-15x+12+2x^2+4x+2=5x^2-5x\Rightarrow -6x=-14\Rightarrow$$

$$\Rightarrow x=\frac{14}{6}=\frac{7}{3}=2\frac{1}{3}$$

31.- $2\left(\frac{x+2}{x-2}\right)-3\left(\frac{x-2}{2x+3}\right)=\frac{x^2+78}{2x^2-x-6}$

Solución:

En primer lugar se factoriza la siguiente expresión:

$$2x^2-x-6=\frac{2(2x^2-x-6)}{2}=\frac{(2x)^2-(2x)-12}{2}=$$

$$=\frac{(2x-4)(2x+3)}{2}=(x-2)(2x+3)$$

Luego:

$$(x-2)(2x+3)\left[\frac{2(x+2)}{x-2}-\frac{3(x-2)}{2x+3}\right]=(x-2)(2x+3)\left[\frac{x^2+78}{(x-2)(2x+3)}\right]\Rightarrow$$

$$\Rightarrow 2(x+2)(2x+3)-3(x-2)^2=x^2+78\Rightarrow 4x^2+6x+8x+12-3x^2+12x-12=x^2+78\Rightarrow$$

$$\Rightarrow 26x=78\Rightarrow x=\frac{78}{26}=3$$

32.- $\dfrac{1}{3x^2+3x-28}-\dfrac{1}{x^2+12x+35}=\dfrac{3}{x^2+x-20}$

Solución:

$$\frac{1}{(x-4)(x+7)}-\frac{1}{(x+5)(x+7)}=\frac{3}{(x+5)(x-4)}\Rightarrow$$

$$\Rightarrow (x-4)(x+5)(x+7)\left[\frac{1}{(x-4)(x+7)}-\frac{1}{(x+5)(x+7)}\right]=(x-4)(x+5)(x+7)\left[\frac{3}{(x-4)(x+5)}\right]\Rightarrow$$

$$\Rightarrow x+5-x+4=3(x+7)\Rightarrow 9=3x+21\Rightarrow x=-\frac{12}{3}=-4$$

33.- $\dfrac{x-2}{x^2+8x+7}=\dfrac{2x-5}{x^2-49}-\dfrac{x-2}{x^2-6x-7}$

Solución:

$$\frac{x-2}{(x+1)(x+7)}=\frac{2x-5}{(x+7)(x-7)}-\frac{x-2}{(x+1)(x-7)}$$

Entonces:

$$(x+1)(x-7)(x+7)\left[\frac{x-2}{(x+1)(x+7)}\right]=(x+1)(x-7)(x+7)\left[\frac{2x-5}{(x-7)(x+7)}-\frac{x-2}{(x+1)(x-7)}\right]\Rightarrow$$

$$\Rightarrow (x-7)(x-2)=(2x-5)(x+1)-(x-2)(x+7)\Rightarrow x^2-9x+14=2x^2-3x-5-x^2-5x+14\Rightarrow$$

$$\Rightarrow -x=-5\Rightarrow x=5$$

34.- $\dfrac{4x+5}{15x^2+7x-2}-\dfrac{2x+3}{12x^2-7x-10}-\dfrac{2x-5}{20x^2-29x+5}=0$

Solución:

En primer lugar se deben factorizar los trinomios denominadores:

(a).-

$$15x^2+7x-2=\frac{15\left(15x^2+7x-2\right)}{15}=\frac{\left(15x\right)^2+7\left(15x\right)-30}{15}=$$

$$=\frac{\left(15x-3\right)\left(15x+10\right)}{15}=\left(5x-1\right)\left(3x+2\right)$$

(b).-

$$12x^2-7x-10=\frac{12\left(12x^2-7x-10\right)}{12}=\frac{\left(12x\right)^2-7\left(12x\right)-120}{12}=$$

$$=\frac{\left(12x+8\right)\left(12x-15\right)}{12}=\left(3x+2\right)\left(4x-5\right)$$

©.-

$$20x^2-29x+5=\frac{20\left(20x^2-29x+5\right)}{20}=\frac{\left(20x\right)^2-29\left(20x\right)+100}{20}=$$

$$=\frac{\left(20x-4\right)\left(20x-25\right)}{20}=\left(5x-1\right)\left(4x-5\right)$$

Ahora:

$$\frac{4x+5}{\left(5x-1\right)\left(3x+2\right)}-\frac{2x+3}{\left(3x+2\right)\left(4x-5\right)}-\frac{2x-5}{\left(5x-1\right)\left(4x-5\right)}=0\Rightarrow$$

$$\Rightarrow\left(5x-1\right)\left(3x+2\right)\left(4x-5\right)\left[\frac{4x+5}{\left(5x-1\right)\left(3x+2\right)}-\frac{2x+3}{\left(3x+2\right)\left(4x-5\right)}-\frac{2x-5}{\left(5x-1\right)\left(4x-5\right)}\right]\Rightarrow$$

$$\Rightarrow\left(4x+5\right)\left(4x-5\right)-\left(2x+3\right)\left(5x-1\right)-\left(2x-5\right)\left(3x+2\right)=0\Rightarrow$$

$$\Rightarrow16x^2-25-\left(10x^2-2x+15x-3\right)-\left(6x^2+4x-15x-10\right)=0\Rightarrow$$

$$\Rightarrow16x^2-25-10x^2+2x-15x+3-6x^2-4x+15x+10=0\Rightarrow$$

$$\Rightarrow-2x-12=0\Rightarrow x=-\frac{12}{2}=-6$$

35.- $\frac{7}{2x+1}-\frac{3}{x+4}=\frac{2}{x+1}-\frac{3\left(x+1\right)}{2x^2+9x+4}$

Solución:

En primer lugar, se busca factorizar el trinomio denominador:

$$2x^2+9x+4=\frac{2\left(2x^2+9x+4\right)}{2}=\frac{\left(2x\right)^2+9\left(2x\right)+8}{2}=$$

$$=\frac{\left(2x+1\right)\left(2x+8\right)}{2}=\left(2x+1\right)\left(x+4\right)$$

El m.c.m. de todos los denominadores es: $\left(2x+1\right)\left(x+4\right)\left(x+1\right)$; entonces, se multiplica ambos lados de la igualdad por el m.c.m.:

$$\left(2x+1\right)\left(x+4\right)\left(x+1\right)\left[\frac{7}{2x+1}-\frac{3}{x+4}\right]=\left(2x+1\right)\left(x+4\right)\left(x+1\right)\left[\frac{2}{x+1}-\frac{3\left(x+1\right)}{\left(2x+1\right)\left(x+4\right)}\right]\Rightarrow$$

$$\Rightarrow 7\left(x+4\right)\left(x+1\right)-3\left(2x+1\right)\left(x+1\right)=2\left(2x+1\right)\left(x+4\right)-3\left(x+1\right)^2\Rightarrow$$

$$\Rightarrow 7\left(x^2+5x+4\right)-3\left(2x^2+3x+1\right)=2\left(2x^2+9x+4\right)-3\left(x^2+2x+1\right)\Rightarrow$$

$$\Rightarrow 7x^2+35x+28-6x^2-9x-3=4x^2+18x+8-3x^2-6x-3\Rightarrow$$

$$\Rightarrow 14x=-20\Rightarrow x=-\frac{20}{14}=-\frac{10}{7}=-1\frac{3}{7}$$

36.- $\dfrac{\left(x+3\right)^2}{\left(x-3\right)^2}=\dfrac{x-1}{x+1}+\dfrac{2\left(7x+1\right)}{x^2-2x-3}$

Solución:

$$\frac{\left(x+3\right)^2}{\left(x-3\right)^2}=\frac{x-1}{x+1}+\frac{2\left(7x+1\right)}{\left(x+1\right)\left(x-3\right)}$$

El m.c.m. de los denominadores es: $\left(x-3\right)^2\left(x+1\right)$; entonces:

$$\left(x-3\right)^2\left(x+1\right)\times\frac{\left(x+3\right)^2}{\left(x-3\right)^2}=\left(x-3\right)^2\left(x+1\right)\left[\frac{x-1}{x+1}+\frac{2\left(7x+1\right)}{\left(x+1\right)\left(x-3\right)}\right]\Rightarrow$$

$$\Rightarrow\left(x+1\right)\left(x+3\right)^2=\left(x-3\right)^2\left(x-1\right)+\left(x-3\right)\times 2\times\left(7x+1\right)\Rightarrow$$

$$\Rightarrow\left(x+1\right)\left(x^2+6x+9\right)=\left(x^2-6x+9\right)\left(x-1\right)+2\left(7x^2-20x-3\right)\Rightarrow$$

$$\Rightarrow x^3+6x^2+9x+x^2+6x+9=x^3-6x^2+9x-x^2+6x-9+14x^2-40x-6\Rightarrow$$

$$\Rightarrow 40x=-24\Rightarrow x=-\frac{24}{40}=-\frac{3}{5}$$

37.- $\dfrac{x-4}{x+5}-\dfrac{x+1}{x-2}=-\dfrac{12\left(x+3\right)}{\left(x+5\right)^2}$

Solución:

$$\frac{(x-4)(x-2)-(x+1)(x+5)}{(x+5)(x-2)}=-\frac{12(x+3)}{(x+5)^2}\Rightarrow\frac{x^2-6x+8-x^2-6x-5}{(x+5)(x-2)}=\frac{-12x-36}{(x+5)^2}\Rightarrow$$

$$\Rightarrow\frac{-12x+3}{(x-2)}=\frac{-12x-36}{x+5}\Rightarrow$$

$$(x+5)(3-12x)=(x-2)(-12x-36)\Rightarrow 3x-12x^2+15-60x=-12x^2-36x+24x+72\Rightarrow$$

$$\Rightarrow -45x=57\Rightarrow x=-\frac{57}{45}=-\frac{19}{15}=-1\frac{4}{15}$$

38.- $\frac{x-3}{x-4}-\frac{x-2}{x-3}=\frac{x+2}{x+1}-\frac{x+3}{x+2}$

Solución:

$$\frac{(x-3)^2-(x-4)(x-2)}{(x-4)(x-3)}=\frac{(x+2)^2-(x+1)(x+3)}{(x+1)(x+2)}\Rightarrow$$

$$\Rightarrow\frac{x^2-6x+9-x^2+6x-8}{(x-4)(x-3)}=\frac{x^2+4x+4-x^2-4x-3}{(x+1)(x+2)}\Rightarrow$$

$$\Rightarrow\frac{1}{(x-4)(x-3)}=\frac{1}{(x+1)(x+2)}\Rightarrow x^2+3x+2=x^2-7x+12\Rightarrow$$

$$\Rightarrow 10x=10\Rightarrow x=\frac{10}{10}=1$$

39.- $\frac{x+6}{x+2}-\frac{x+1}{x-3}=\frac{x+5}{x-1}-\frac{x}{x+4}$

Solución:

$$\frac{(x+6)(x-3)-(x+2)(x+1)}{(x+2)(x-3)}=\frac{(x-5)(x+4)-x(x-1)}{(x-1)(x+4)}\Rightarrow$$

$$\Rightarrow\frac{x^2+3x-18-x^2-3x-2}{(x+2)(x-3)}=\frac{x^2-x-20-x^2+x}{(x-1)(x+4)}\Rightarrow$$

$$\Rightarrow\frac{-20}{(x+2)(x-3)}=\frac{-20}{(x-1)(x+4)}\Rightarrow x^2+3x-4=x^2-x-6\Rightarrow$$

$$\Rightarrow 4x=-2\Rightarrow x=-\frac{1}{2}$$